ALGEBRA

List 6.

Change of a basis. Linear transformations: matrix, kernel, range

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1. Is $(1,-1,2),(2,1,0),(2,0,-1)$ a basis in $\mathbb{R}^3$? If yes, give the coordinates of the vector $(2,3,4)$ in this basis.

2. Is $(1,-1,1),(1,1,0),(0,1,1)$ a basis in $\mathbb{R}^3$? If yes, give the coordinates of the vector $(-2,0,3)$ in this basis.

3. Is $(1,-1,1),(1,1,0),(1,0,1)$ a basis in $\mathbb{R}^3$? If yes, give the coordinates of the vector $(-3,0,2)$ in this basis.

4. Is $(1,-1,1),(1,1,0)$ a basis in $\mathbb{R}^3$? If yes, give the coordinates of the vector $(-2,0,2)$ in this basis.

5. Is $(-2,0,2)$ a linear combination of $(1,-1,1),(1,1,0)$? If yes, with which coefficients?

6. Is $(-2,0,1)$ a linear combination of $(1,-1,1),(1,1,0)$? If yes, with which coefficients?

7. Let the linear mapping of $\mathbb{R}^2$ be given by $T(x,y)=(2x+y,x-y)$. Find its matrices in the standard basis $B=\{e_1,e_2\}$ and in the basis $B'=\{v_1,v_2\}$ given by $v_1=(1,1),v_2=(1,-1)$.

8. The linear mapping of $\mathbb{R}^2$ transforms the vector $(1,2)$ to $(-1,1)$, and the vector $(2,1)$ to $(3,1)$. Write the matrix of this mapping in the standard basis in $\mathbb{R}^2$.

9. For the linear mapping of $\mathbb{R}^2$ which corresponds to rotation clockwise around the origin by the angle α composed with the reflection with respect to Ox axis, write the matrix of this mapping in the standard basis in $\mathbb{R}^2$.

10. For the linear mapping of $\mathbb{R}^2$ which corresponds to reflection with respect to

(a) the Oy axis;

(b) the line $y+x=0$;

(c) the line $3y-4x=0$,

rite the matrices of these mappings in the standard basis in $\mathbb{R}^2$.

11. For the linear mapping of $\mathbb{R}^3$ which corresponds to reflection with respect to

(a) the Oz axis;

(b) the Oyz plane;

(c) the plane $x+2y-3z=0$,

write the matrices of these mappings in the standard basis in $\mathbb{R}^3$.

12. For the linear mappings of $\mathbb{R}^3$ which corresponds to rotation counter-clockwise around the Oy and Oz axes by the angle α, write the matrices of these mappings in the standard basis in $\mathbb{R}^3$. For which values of α these mappings commute?

13. Write the matrices in the standard basis in $\mathbb{R}^3$ of the rotation counter-clock wise by angle $\frac{2\pi}{3}$ around the line $x=y=z$.